BASIC FORMULATION OF ENVIRONMENTAL DESIGN, MAINTENANCE AND REPAIR OPTIMIZATION OF CONCRETE STRUCTURES

Petr Štěpánek **Jan Plšek**

Summary

Basic formulation of environmental optimization problem of concrete structures design is presented. The developed theory is documented with solution of deterministic and probabilistic optimization design of cross section loaded by bending moment.

Keywords: Optimization, design, concrete structure, environmental indicators, embodied CO_2 and SO_2 exhalations

1 Introduction

The obvious aim of structural engineering design is to achieve a design that meets all requirements (formed as restrictive conditions named bounds and constraints) in the most appropriate way, usually at the lowest cost (measured by the objective function value), as described, among others, by Frangopol and Iizuka [1]. The simplest but most often used approach is the trial and error method (if more than one trial is performed); furthermore, the designer is usually satisfied with a solution that is feasible and the best of the tested designs, rather than the overall best one. However, real-world structural optimisation does not deal with elementary problems, and hence, common sense reasoning does not, in general, lead to acceptable designs. Therefore, the request to attain qualitatively advanced structural design requires the support of computers as well as optimisation theory, such as proposed by Brousse [2].

2 Optimised design formulation for concrete structures

To find the best possible design without the reliability system decrease as the whole, it is necessary to use the optimised method. The optimisation methods and theories of design had significantly developed in the last thirty years. However, it must be said that a huge part of the optimised methods is specialized in the area of the structure design considering all variables as deterministic (DBSO – Deterministic Based Structure Optimisation). The general deficiency of the DBSO is an omission of uncertainties at determining loads, material characteristic, response and stability of the structure. Elimination of the deficiency enables the combination of the design methods based on the comprehensive probabilistic accesses and optimised methods called Reliability Based Structure Optimisation (RBSO).

2.1 Reliability based optimization (RBSO) – consideration of randomness of some input variables

The task is defined as follows
a) the target function reaches the extreme

$$\{f(\{A_s\})\} = \text{ extreme,} \tag{1.a}$$

under keeping the restrictive conditions implicit from
b) the requirements of reliability expressed in probabilities

$$P_{fj}(\{A_s\}) \le P_{fj}^0 \qquad j = 1, ..., np, \tag{1.b}$$

and others conditions in which may be included the probabilities. These conditions can be expressed in a form of
c) the equalities

$$\{h(\{A_s\})\} = \{0_1\}, \tag{1.c}$$

d) the inequalities

$$\{g(\{A_s\})\} \le \{0_2\}, \tag{1.d}$$

where $\{A_s\} = \{A_{s1},...,A_{snt}\}^T$ are the design variables, $\{f(\{A_s\})\} = \{f_1(\{A_s\}),..., f_t(\{A_s\})\}^T$ is vector of the target functions, $f_i(\{A_s\})$ is i-th target function, $\{h(\{A_s\})\}$ is vector of the restrictive conditions in a form of the equations, $\{g(\{A_s\})\}$ is vector of the restrictive conditions in a form of inequalities, $P_{fj}(\{A_s\})$ is probability of the structure failure (j-th condition of the reliability relating to some mode of the failure or to the applicability in some locality), P_{fj}^0 is permissive probability of the failure determined for j-th conditions of the reliability, $\{0_1\}$ and $\{0_2\}$ are zero vectors of the relevant type.

For established vector of the design variables, the conditions of equilibrium of solved structure – with application of discretion of the solved task by a FEM method – is possible to express in the form (1.b), where

$$\{h(\{A_s\})\} = [K(\{A_s\})]\{\Delta\} - \{F\}, \tag{2}$$

$[K(\{A_s\})]$ is stiffness matrix of solved structure, $\{\Delta\}$ is vector of nodal parameter of deformation and $\{F\}$ is loading vector of a structure.

2.2 Deterministic based optimization (DBSO)

The task is defined analogously to the RBSO but the conditions (1.b) are not used and in the relations (1.a), (1.c) and (1.d) the probabilities are not included. For solution of this task it is possible to apply mathematical programming algorithms and the algorithms based on evolution processes too.

2.3 Environmental aspects of design

Environmental impact assessment of design, maintenance and/or repair of concrete structure examines the influence of projected works on the major categories of environmental problems: land quality, air and water pollution, global environmental concerns (specifically greenhouse gas emissions and protection of the ozone layer).

The method of assessing the environmental impact of a concrete structure is connected with the life cycle of the whole structure. A Life Cycle Assessment (LCA) can be used in two ways:

- To determine the total environmental impact of structure or its design alternatives with the aim of comparing them. For a designer, an LCA can provide information if he/she has to choose among design alternatives or among different components or materials.
- To determine the most important causes of a product's environmental impact. A designer can concentrate on achieving improvements based on LCA

A designer wishing to use LCA in the design process is faced with two major problems:

- It is difficult to interpret the result of LCA. Within an LCA, it is possible to determine the environmental impact of a product to the greenhouse effect, acidification and other environmental problems in the life cycle, while the total environmental impact remains unknown. The reason is the lack of mutual weighting method for environmental effects (indicators).
- In general, the careful collection of all environmental data in a life cycle of product/plan is complex, expensive and time consuming.

For checking of environmental impact of structure could be used the environmental impact indicators (EII). The functions EII are **descriptive** in nature, for example:

- to describe and diagnose the existing situation as regards sustainability, and
- to forecast future trends relating to sustainable development;

and **normative**, for example:

- to assess the existing situation and trends against the background of qualitative and quantitative objectives for sustainable development
- to assist with formulating objectives of sustainability more precisely and quantifying them
- to facilitate policy decision-making
- to monitor the success of sustainability-related measures (What does this mean?) for example: it would be possible to monitor the influence of some standards on sustainability
- to contribute to public awareness and communication.

The optimisation problem of the concrete structure "sustainment" may be, e.g. by [3], expressed by the targed function:

$$\{f(\{A_s\})\} = f(\min E_{tot}, \min C_{tot}, \max S_{tot}) \tag{3}$$

where E_{tot} is the gross environmental impact, C_{tot} is the gross cost and S_{tot} is the gross social-cultural quality.

The multi-criterion problem is possible to convert to the mono-criterion one by the weight implementation among the individual criteria but it is evident that at the different weights given to the individual criteria, obtained optimums can differ. Furthermore, it is not guaranteed that at the searching for optimum by the different method of multi-criterion optimisation (selection strategy), the given method will always converge.

2.3.1 Environmental impact

The magnitude of the environmental impact of the structure E_{tot} may be expressed

$$E_{tot} = \sum p_i E_i \tag{4}$$

where p_i is the probability of incidence of i-th environmental impact E_i. Relation (4) may be explicated as

$$E_{tot} = E_{ini} + E_{oper} + E_m + \sum p_f E_{repair} + \sum p_{renov} E_{renov} + E_{demol} + E_{recycl} \ , \tag{5}$$

where the individual environmental impacts represent: E_{ini} the impact connected with the materials and members production and to the design and realisation of the construction, E_{oper} the impact connected with the operation of the construction, E_m the impact connected with maintenance, E_{repair} the impact connected with the repair of failures, E_{renov} the impact connected with the reconstruction, E_{demol} the impact connected with the demolition, E_{recycl} the impact connected with the recycling and p_f , p_{renov} are the referring probabilities of possible failures origin and reconstructing interventions.

These partial environmental impacts (generally denoted E_i) related to the individual phases of the concrete structure life cycle should include every important environmental impacts that correspond to the decisive environmental criteria and are expressed as

$$E_i = \sum w_j Q_j \ , \tag{6}$$

where $\{w_j\} = (w_1 \ ... \ w_m)^T$ is the weight vector expressing importance of the individual criteria, $\{Q_j\} = (Q_1 \ ... \ Q_m)^T$ is the vector of embodied values relating to the individual criteria, m is the number of critera.

2.3.2 Cost of construction

The gross cost of construction C_{tot} is expressed as a function expressing the construction cost during its life cycle, i.e.

$$C_{tot} = \sum r_i \ C_i \tag{7}$$

or in the more detailed form as:

$$C_{tot} = C_{ini} + C_{oper} + C_m + \sum r_f C_{repair} + \sum r_{renov} C_{renov} + C_{demol} + C_{recycl} \tag{8}$$

where C_{ini} is the construction initial cost (i.e. a sum of the costs of materials, transport, design and construction), C_{oper} is the operating cost, C_m is the maintenance cost, C_{repair} is the repairs cost, E_{renov} is the accidental reconstruction (renovation) cost, C_{demol} is the demolition cost, C_{recycl} is the recycling cost and r_f , r_{renov} are probabilities of the possible failures origin and reconstructing interventions.

2.3.3 Social and cultural quality

The specification of this function and quantification of the individual criteria represents very soft problem. It is possible that its introducing could lead to the confusion and devaluation of the optimisation results performed on the criteria according to 2.3.1, 2.3.2.

The first two research points were completed and all the results were analysed [2]. Choice of the most suitable type of reinforcement was achieved based on obtained results. The best cohesion with the concrete, material properties and demand factor of the production of the reinforcement and the surface preparation were confronted. All these parameters influence the price and the efficiency of the developed reinforcement.

3 Numerical example

Let a simply supported beam be uniformly loaded all along its span l by its own weight g and imposed load q; the beam is of constant RC rectangular cross-section. The objective is to design all unknown cross-section parameters with help of the RBSO and DBSO algorithms and to compare the acquired results. Some of the parameters are given: strength classes of materials (f_c and f_s), number of reinforcement bars $N_ø$, and their positions in the cross-section (covers c_1 to c_4, see **Fig. 1**). Reinforcement bar diameters ø and cross-section dimensions b and h (width and height, respectively) are left for optimisation. The calculations are carried out in 5 variants of weighted sums α_p, α_{co}, α_{so} see (9) and 2 variants of cross-section dimensions, which are either optimised or given (fixed).

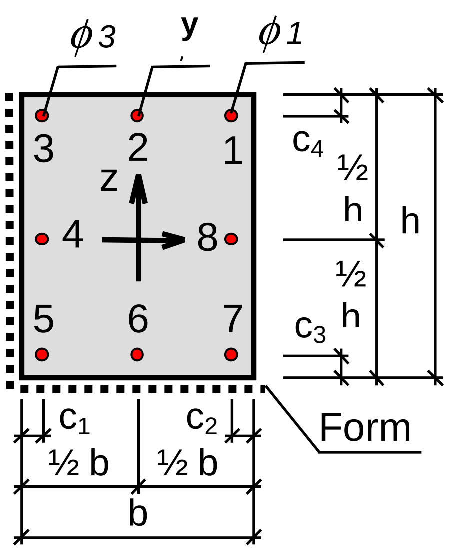

Fig. 1 Cross-section

For the purpose of comparing DBSO and RBSO, probabilities are omitted from the RSBO objective function, however, they are still involved in constraints, and a common simple objective function is defined. We are interested in economic and ecological aspects (acquisition costs and CO_2 and SO_2 emissions associated with concrete member formation, respectively). The problem is a multi-criteria one and all the mentioned aspects should be minimised. The method of weighted sums was applied to obtain a single objective function. However, the terms of the objective function have different units and thus cannot be directly summed, and for that reason they must be first normalized by chosen reference values:

$$f(x) = \alpha_p P(x) / {}^0P + \alpha_{co} CO_2(x) / {}^0CO_2 + \alpha_{so} {}^0CO_2(x) / {}^0SO_2 \, , \qquad (9)$$

where the symbols used mean: $P = V_c U^c_p + m_s U^s_p + S_f U^f_p$, $CO_2 = V_c U^c_{co2} + m_s U^s_{co2}$, $SO_2 = V_c U^c_{so2} + m_s U^s_{so2}$ are acquisition costs, amount of CO_2 and SO_2 emissions; ${}^0P$, ${}^0CO_2$, ${}^0SO_2$ are reference values set by problem designer (constants); α_p, α_{co}, α_{so} are weights in the objective function for P, CO_2, and SO_2, respectively; V_c, m_s, S_f are concrete volume, steel weight and form area of a 1m long beam fragment with a designed cross-section, respectively; U^c_p, U^s_p, U^f_p are unit costs of concrete, steel and form, respectively (concrete and steel costs depend on design variables f_c and f_s); U^c_{co2}, U^s_{co2} are unit amounts of CO_2 emissions from concrete and steel; respectively U^c_{so2}, U^s_{so2} are unit amounts of SO_2 emissions from concrete and steel, respectively.

Given cross-section parameters are: concrete C25/30, steel B490(R), number of reinforcement bars $N_ø = 8$ and axial covers $c_1 = .. = c_4 = 40$ mm. Design variable bounds are: cross-section dimensions: b, h = <200, 600> mm; reinforcement bar diameters $ø_1$, $ø_3$,

$ø_5$, $ø_7$ = <10, 22> mm and $ø_2$, $ø_4$, $ø_6$, $ø_8$ = <0, 22> mm; loading case specifications: number of loading cases $n_L = 1$ (a simply supported beam, $My(\xi) = (q(\xi) + b\ h\ \gamma_c)\ l^2 / 8$, where l is the effective span (8 m), γ_c is the reinforced concrete unit weight (25 kN/m^3) and $q(\xi)$ is the given characteristic value of imposed load (14 kN/m). The variant with fixed cross-section dimensions b = 300 mm, h = 500 mm was solved too. Each of the random quantities is assumed to be normally distributed without mutual correlations. The mean value of $q(\xi)$ is determined in such a way that value q_k is its 95 % fractile. Multipliers γ_E, γ_g = 1,35; γ_q = 1,5 are relevant partial factors for permanent and variable loads in accordance with EN1990 (2004).

All parameters needed in the objective function are: U^c_p = 83,29 €/ m^3, U^s_p = 0,675 €/kg, U^f_p, = 14,22 €/ m^2, U^c_{co2} = 321 kg/m^3, U^s_{co2} = 767 kg/t, U^c_{so2} = 1,11 kg/m^3, U^s_{so2} = 3,67 kg/t.

Five alternatives α_p = 1,0 (0,75; 0,5; 0,25; 0,0), α_{co}= 0,0 (0,125; 0,25; 0,375; 0,5), α_{so} = 0,0 (0,125; 0,25; 0,375; 0,5) of weights in the objective function were taken into account.

ULS strain limits for steel are $\varepsilon^{St}_{\min} = -3{,}5.10^{-3}$, $\varepsilon^{St}_{\max} = 10.10^{-3}$ and those for concrete are $\varepsilon^{C}_{\min} = -3{,}5.10^{-3}$, $\varepsilon^{C}_{\max} = \infty$. The required cross-section reliability is given by the minimal Cornell safety index $\beta_{\min} = 3{,}8$, which corresponds to the maximal failure probability $p_f^{\max} \approx 7{,}2.10^{-5}$. The number of simulations for p_f estimation is set to $N_S = 100$. $Z(\xi)$ is assumed to be normally distributed. The proportion of b to h is constrained by a lower bound of 0,4. The cross-section is kept vertically symmetrical, so additional constraints must be set up for bar diameters.

Tab. 1 Results of DBSO (all variants "b, h = const" led always to same results shown in first column; reliability indexes for DBSO were calculated additionally after termination of optimisation process)

Weights / Dimensions		Weights 1-5 Fixed	Weights 1 Optimised	Weights 2 Optimised	Weights 3 Optimised	Weights 4 Optimised	Weights 5 Optimised
$\phi_1 = \phi_3$		10.0	10.0	10.0	10.0	10.0	10.0
$\phi_5 = \phi_7$		21.5	22.0	22.0	22.0	22.0	22.0
ϕ_2	[mm]	0.0	0.0	0.0	0.0	0.0	0.0
$\phi_4 = \phi_8$		0.0	0.0	6.1	6.3	6.6	0.0
ϕ_6		20.9	22.0	22.0	22.0	22.0	22.0
Width b	[mm]	300	200	200	200	200	200
Height h	[mm]	500	454	446	446	445	454
Steel Area	[mm^2]	1222	1297	1356	1361	1366	1297
Concrete Area	[mm^2]	150000	90752	89201	89105	88991	90752
Form Length	[mm]	1300	1108	1092	1091	1090	1108
Cost of Steel	[EUR]	6.48	6.88	7.19	7.21	7.24	6.88
Cost of Concrete	[EUR]	12.49	7.56	7.43	7.42	7.41	7.56
Cost of Form	[EUR]	18.49	15.75	15.53	15.51	15.50	15.75
Total Cost	[EUR]	37.46	30.19	30.15	30.15	30.15	30.19
Amount of CO_2	[kg]	55.510	36.943	36.800	36.795	36.788	36.943
Amount of SO_2	[kg]	0.202	0.13811	0.13809	0.13810	0.13812	0.13811
Target function	[-]	0.846	0.681	0.655	0.63	0.605	0.581
Reliability Index	[-]	3.8	3.8	3.8	3.8	3.8	3.8

The numerical results are listed in **Tab. 1** and **2**. The variant with fixed cross-section dimensions produced the same results for DBSO without regard to summation weight changes, thus all five cases are written in a single column, similarly to RBSO. The amount of steel computed by DBSO is always greater than that obtained by RBSO, and the amount of concrete is almost equal in all cases. Modifications of summation weights caused only small changes in the CO_2 and SO_2 emission results. There are substantial differences in the reliability indexes (on average 17 % for "optimised dimensions" cases) and total costs (on average 22 % for "optimised dimensions" cases).

Tab. 2 Results of RBSO (all variants "*b*, *h* = const" led always to same results shown in first column; minimal required reliability index was 3,8; it was the main restrictive condition of the optimisation)

Weights		Weights 1-5	Weights 1	Weights 2	Weights 3	Weights 4	Weights 5
Dimensions		Fixed	Optimised	Optimised	Optimised	Optimised	Optimised
$\phi_1 = \phi_3$		10.0	17.1	16.5	22.0	21.9	22.0
$\phi_5 = \phi_7$		22.0	22.0	22.0	22.0	22.0	22.0
ϕ_2	[mm]	0.0	14.8	22.0	15.8	20.6	21.5
$\phi_4 = \phi_8$		7.1	17.3	19.0	20.4	21.7	22.0
ϕ_6		22.0	22.0	22.0	22.0	22.0	22.0
Width b	[mm]	300	254	244	235	228	227
Height h	[mm]	500	424	406	392	381	378
Steel Area	[mm^2]	1376	2244	2516	2753	2967	3024
Concrete Area	[mm^2]	150000	107693	99038	92326	86878	85507
Form Length	[mm]	1300	1102	1056	1020	989	982
Cost of Steel	[EUR]	7.29	11.90	13.34	14.60	15.73	16.04
Cost of Concrete	[EUR]	12.49	8.97	8.25	7.69	7.24	7.12
Cost of Form	[EUR]	18.49	15.66	15.02	14.50	14.07	13.96
Total Cost	[EUR]	38.27	36.53	36.61	36.79	37.04	37.11
Amount of CO_2	[kg]	56.433	48.081	46.937	46.213	45.752	45.657
Amount of SO_2	[kg]	0.206	0.18419	0.18241	0.18180	0.18191	0.18204
Target function	[-]	0.877	0.825	0.808	0.788	0.766	0.742
Reliability Index	[-]	4.38	4.51	4.48	4.47	4.43	4.42

4 Conclusions

From the related theory of optimization and stochastic programming, it follows that in most cases that involve random parameters, the stochastic approach is more advantageous than the deterministic approach based on the use of expectations. The particular conclusions related to the application area and computed example follow.

- Materially-optimised structures seem to be environmentally-friendly without there being any explicitly prescribed goal of taking into account ecological aspects. This is because material savings mean less pollution.
- RBSO is more flexible in taking into account the varying contributions to the failure probability (according to the particular case) created by constituent random elements. Partial factors are fixed constants without regard to a particular case and were assigned in a more or less transparent way. Therefore RBSO is a safer method than DBSO.
- From the results of the numerical example, it follows that DBSO, in comparison to RBSO, results in over-designed cross-sections.

- With the help of RBSO, it is possible to achieve a structural design with a well-proportioned level of failure probability (with respect to various failure modes), therefore the structure is economically and environmentally advantageous.

This contribution has been prepared with the financial support of the Ministry of Education, Youth and Sports project No. 1M06005. The theoretical results were partially gained from project GAČR 103/07/0400 and from project MPO CR No. 1HPK2/57.

References

[1] BROUSSE, P. (1988). "*Optimisation mechanics: problems and methods.*" North Holland, ISBN 0-444-704-949.

[2] FRANGOPOL, D. M., IIZUKA M. (1992). "Probability-based structural system design using multicriteria optimization." *In: Proceedings of the 4th AIAA/USAF/NASA/OAI Symposium on Multidisciplinary Analysis and Optimization*, AIAA-92-4788-CP Paper, Part 2, Cleveland, Ohio, 794-798.

[3] PLŠEK, J., ŠTĚPÁNEK, P. (2005b). "Optimisation of reinforced concrete cross-sections subjected to spatial load." *In: Reliability, Safety and Diagnostics of Transport Structures and Means,* Pardubice 7-8 July 2005, CZ, University of Pardubice, 280-287, ISBN 80-7194-769-5.

Prof. RNDr. Ing. Petr Štěpánek, CSc.
✉ VUT v Brně, Fakulta stavební
Veveří 95
602 00 Brno
☎ 541 147 848
🗐 549 250 218
☺ stepanek.p@fce.vutbr.cz
URL www.fce.vutbr.cz

Ing. Jan Plšek
✉ VUT v Brně, Fakulta stavební
Veveří 95
602 00 Brno
☎ 541 141 111
🗐 549 250 218
☺ plsek.j@fce.vutbr.cz
URL www.fce.vutbr.cz